Revista Cuadernos Administracion | Vol. 42 | Issue 84 | Jan. – Apr. | 2026 | e20414855
https://doi.org/10.25100/cdea.v42i84.14855
Article of Scientific and Technological Research

Multi-criteria ABC classification and mathematical modeling for multi-product inventory control with budget constraints

Clasificación ABC multicriterio y modelación matemática para el control de inventarios multi producto con restricciones de presupuesto

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Abstract

The multi-item inventory management problem with stochastic demand and budget constraints is a challenge for managers, as they must decide when to place an order and what quantity to order. The decision to order depends on whether the current inventory is sufficient to meet future demand, and the quantity to order depends on the estimate of future demand and the desire to have a permissible amount of stockouts. Due to demand uncertainty, stock replenishment decisions are expected to provide a high level of service for all items; however, if stockouts occur, they are expected to have a minor impact on customers. To solve this problem, a hybrid model is proposed that combines multi-criteria classification to classify items according to their relative importance with a linear programming model that establishes replenishment quantities, considering the possibility of different service levels. The results show, for a case study in a healthcare company that must jointly manage a total of 217 items, that the assigned budget allows balancing the purchase quantities of each item under high service levels.

Key Words: Multi product, Inventory Control, TOPSIS, Mathematical Model.

Resumen

El problema de gestión de inventarios para múltiples ítems con demanda estocástica y límites de presupuesto es un desafío para administradores en la medida de que deben decidir cuándo emitir una orden y qué cantidad ordenar. La decisión de ordenar depende de considerar si el inventario actual es el suficiente para atender la demanda futura y la cantidad a ordenar depende de la estimación de la demanda futura y el deseo de tener una cantidad permisible de productos agotados. Debido a la incertidumbre de la demanda, se espera que las decisiones de reposición de existencias proporcionen un alto nivel de servicio para todos los artículos; sin embargo, si se producen roturas de stock, se espera que estas tengan un impacto menor en los clientes. Para resolver esta problemática, se propuso un modelo híbrido que combina la clasificación Multicriterio para clasificar los ítems de acuerdo con su importancia relativa con un modelo de programación lineal que establece las cantidades a reponer considerando la posibilidad de tener diferentes niveles de servicio. Los resultados muestran para un caso de estudio en una empresa del sector salud que debe gestionar de forma conjunta un total de 217 ítems,

Thematic lines: Management and Organizations.

JEL code: C61, C44..

Submitted: 23/04/2025
Reviewed: 13/09/2025
Accepted: 02/07/2026
Published: 15/09/2026

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How to cite this paper?
Londoño Ortega, J. C. (2026). Multi-criteria ABC classification and mathematical modeling for multi-product inventory control with budget constraints. *Cuadernos de Administración*, *42*(84), e20414855. https://doi.org/10.25100/cdea.v42i84.14855

que el presupuesto asignado permite balancear las cantidades a comprar de cada ítem bajo altos niveles de servicio.

Palabras clave: Múltiples productos, Control de Inventarios, TOPSIS, Modelación matemática.

Introduction

In inventory management, managers must decide, for multiple products, when to review, when to place an order and what quantity to order under conditions of stochastic demand, to keep them available at the lowest possible operating cost. This task is rather complex, since some of the items come from the same supplier or the same geographical location. In certain cases, this management is undertaken by considering the possibility of securing discounts, taking advantage of economies of scale or accommodating constraints such as transport capacity, storage capacity or capital availability.

To meet stochastic demand, safety stocks are held; these are intended to cover any demand exceeding an established average that occurs during the replenishment period. This strategy entails two scenarios: firstly, that inventory levels rise unnecessarily, leading to excess stock when demand is low; and secondly, that shortages arise when demand exceeds inventory levels. As a result, managers strive to set inventory levels by taking into account the probability of a given demand occurring. Regarding demand coverage, the service level indicators P_1 and P_2 are considered. These represent the outcomes of meeting demand either in terms of frequency (occasions) or quantity fulfilled for each respective indicator, as detailed in Nahmias (2008).

On the other hand, issuing an order inherently incurs a range of costs, such as administrative processes, bid evaluations, transportation expenses, and so forth. Consequently, the aim is always to minimise the number of orders placed to avoid these costs becoming excessively high. However, reducing the number of orders necessitates increasing the quantity ordered per product, which raises the capital invested and, in turn, leads to higher inventory holding costs. These holding costs are determined by various factors, including the cost of capital, storage, product deterioration, insurance, and taxes.

Among the most critical real-world constraints are those linked to budgetary limitations, which set a maximum amount of money that can be invested in inventories (Hadley & Whitin, 1963). In response to budget constraints, “quick response” strategies emerged in the 1990s. One such approach involves placing two orders within a single demand period. The first order secures products at a lower price, while a second order is placed closer to the end of the season - when potential demand is better understood - albeit with the risk of higher product prices (Serel, 2012).

In the joint control of multiple products, two primary review systems are fundamentally considered: Continuous Review and Periodic Review.

In continuous review, it is assumed that all items are monitored continuously, and a reorder point is established. If any item reaches its reorder point, a decision is made on which other products to order.

In periodic review, a fixed review period is set during which a predefined group of products is jointly assessed and ordered simultaneously under predetermined criteria.

In addition to the above, there is a relative importance of each item within the organisation. This importance is determined based on established criteria, typically resulting in the grouping of items into distinct categories. These categories are used by managers to define inventory control policies, which aim to establish how each item will be managed to meet demand under a defined service level while keeping ordering and holding costs under control.

Regarding the relative importance of each item, this may be established depending on the context. For example: In commercial mass-consumption products, importance may be defined by revenue generated over a specific period or via traditional ABC classification based on the Pareto principle (Silver et al., 1998); for spare parts, importance could relate to their criticality during equipment failure, potential for rapid repair, or other criteria (Roda et al., 2014); for healthcare products, importance may be tied to life-saving capacity or improvements in patients' quality of life (Partovi & Burton, 1993). The Flexible and Interactive Tradeoff method was proposed for the ABC classification of a total of 48 medicines, the model used five criteria (cost, demand, lead time, volume and criticality) and one structure based on two relations *Preference* and *Indifference* and apply two mathematical model to assign the relative importance for each relation. The results classified the medicines into three classes: W, B and M, where items classified as W are more critical and must be reviewed weekly, items classified as B are less critical and must be reviewed biweekly, and items classified as M have low criticality and must be reviewed monthly (de Assis et al., 2022).

The combined problem of multiproduct ABC classification with inventory management for items with stochastic demand, to my knowledge, has been studied little. The literature review with the keywords Classification ABC Multicriteria, Mathematical Programming, Inventory Control and Budget constraint found very few publications. For example, in Yang et al. (2017) a mixed integer linear programming model is presented whose objective is to maximise the Net Present Value for the control of multiple products with a predetermined ABC classification. This model considers, among other restrictions, capital restrictions. However, here, the need to avoid out-of-stocks is not discussed, that is, to improve the level of service. Another case is the one presented in Abdolazimi et al. (2021), the authors develop an integrated mathematical model of ABC classification under the criteria of cost and benefit reduction, whose objective function is to maximise the benefit of the items in inventory; Additionally, they consider budget restrictions and service levels. Finally, Ly & Raweewan (2021) present a mathematical model for inventory control considering budget and space limitations in a warehouse; the model optimises the classification of inventories and determines the optimal level of each class; results are compared with results obtained through other classifications.

This work presents a methodology for a healthcare-sector company that provides medication dispensing services with stochastic demand and budget constraints over a monthly planning horizon. The methodology is hybrid and unfolds in two phases:

- First phase: Items are classified according to their relative importance.
- Second phase: Using the results from the first phase, a mathematical model is developed to define a replenishment policy that determines order quantities without exceeding the allocated budget.

Methodology

The purpose of this methodology is to determine the quantities of products to order during the period, maximizing the use of the available budget while considering current inventory levels, budget constraints, and the relative importance of each item. Figure 1, illustrates the two phases of the methodology. The *First phase* classifies the different items based on various criteria, grouping them into three categories according to their importance, following the Pareto principle. This classification is obtained using the TOPSIS technique. The *Second phase* is developed through mathematical modelling. The objective here is to maximize the quantity of products to acquire by defining the service level policy. To this end, three models were proposed in which the decision variable corresponds to the number of standard deviations to be assigned in the calculation of safety stock.

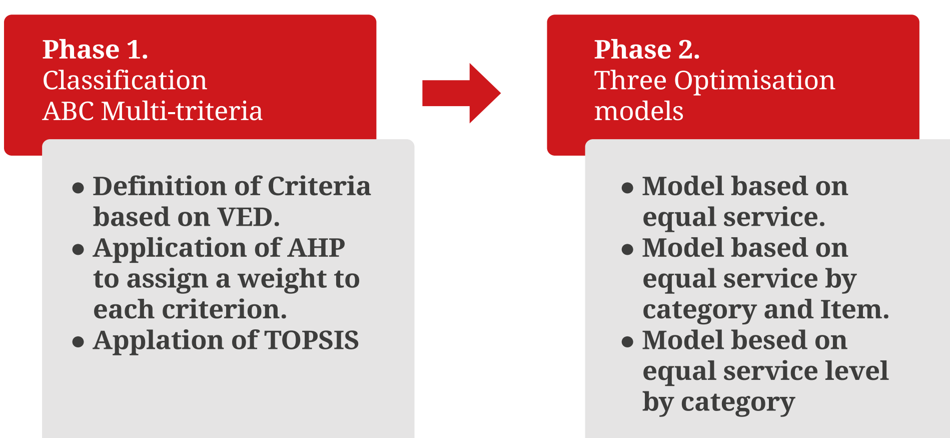

Figure 1. Methodological phases to multi-product inventory control method with budget constraints

Phase 1. Classification ABC Multi-criteria

In the healthcare sector, there is a wide variety of pharmaceutical products with distinct physical characteristics, designed to fulfil specific roles in patient care. Managing these products requires significant logistical effort, compounded by the perceived relative importance assigned by decision-makers responsible for stock replenishment. To streamline inventory management, an ABC classification is applied, which involves defining criteria to assess the relative importance of each item.

Traditionally, ABC classification focuses on two criteria: Demand and Cost. However, given the altruistic mission of healthcare-sector organisations, additional criteria were incorporated. This included the VED (Vital, Essential, Desirable) classification, which categorises medicines based on their criticality (Kumar & Chakravarty, 2015).

Using the combined criteria of Demand, Cost, and VED classification, the TOPSIS multi-criteria method was applied to determine the relative importance of each item.

The TOPSIS method identifies solutions closest to an ideal solution by applying a distance metric (Hwang et al., 1993). The procedure for TOPSIS is as follows:

Step1. Decision Matrix

This matrix is divided into two parts:

- A vector contains the weighting W_j for each of the criteria.
- A set of alternatives (pharmaceutical products) C_i for each of the ii alternatives, along with the rating X_{ij} for each criterion. Table 1 shows the decision matrix.

The weights W_j can be obtained from a predefined scale, by means of a multicriteria technique, or set by management.

Table 1. Decision Matrix of the TOPSIS method

Relative Importance (Weights)	W_1	W_2	W_3
Alternative \ Criteria	C_1	C_2	C_3
A_1	X_{11}	X_{12}	X_{13}
A_2	X_{21}	X_{22}	X_{23}
A_3	X_{31}	X_{32}	X_{33}
⋮	⋮	⋮	⋮
A_n	X_{41}	X_{42}	X_{43}

Step 2. Construction of the normalised matrix

Since the values obtained in the relationships between alternatives and criteria are not expressed in the same units for all criteria, the decision matrix is normalised by means of Equation 1.

$$r_{ij} = X_{ij}/\sqrt{(\sum X_{ij}^2)}\ \forall\ i = 1, \ldots, m; j = 1, \ldots, n \tag{1}$$

Where, X_{ij} is the value assigned to each alternative for each criterion, and r_{ij} denotes this normalised value.

Step 3. Construction of the weighted normalised matrix

The weighted normalised matrix is obtained by multiplying each value in the normalised matrix by its corresponding W_j, using Equation 2.

$$v_{ij} = w_j\, r_{ij} \tag{2}$$

Step 4. Determination of the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS)

The Positive Ideal Solution is obtained by selecting the best values for each criterion from the weighted normalised matrix, using Equation 3.

$$A^{+} = \left\{v_1^{+}, \ldots, v_n^{+}\right\} \tag{3}$$

These optimal values are presented in Equation 4.

$$v_1^{+} = \left\{max(v_{ij})\ si\ j \in\ J;\ min(v_{ij})\ si\ j \in\ J'\right\} \tag{4}$$

The Negative Ideal Solution is obtained by selecting the worst values for each criterion from the weighted normalised matrix, using, Equation 5 .

$$A^{-} = \left\{v_1^{-}, \ldots, v_n^{-}\right\} \tag{5}$$

These optimal values are presented in Equation 6

$$v^{-} = \{min(v_{ij})\ si\ j \in J;\ max(v_{ij})\ si\ j \in J'\} \quad (6)$$

Where j is associated with the benefit criteria and J' with the cost criteria.

Step 5. Calculation of the separation measures for each alternative

The separation from the positive ideal alternative is obtained from Equation 7.

$$S_i^{+} = \left[\sum (v_i^{+} - v_{ij})^2\right]^{1/2} \quad i = 1, \ldots, m \quad (7)$$

The separation from the negative ideal alternative is obtained from Equation 8.

$$S_i^{-} = \left[\sum (v_i^{-} - v_{ij})^2\right]^{1/2} \quad i = 1, \ldots, m \quad (8)$$

Step 6. Calculation of the relative closeness to the ideal solution

The set of alternatives can be ranked in descending order of the closeness coefficient C_i^*; the higher the coefficient, the greater the priority of the $i\ th$ –corresponding alternative Equation 9.

$$C_i^* = \frac{S_i^{-}}{S_i^{+} + S_i^{-}},\ 0 < C_i^* < 1 \quad (9)$$

Mathematical Model for Determining Order Quantities

Given the presence of a budget constraint, it may not be possible to purchase all items for a desired service level, that is, to hold a safety stock that aligns with a policy aiming to satisfy the target demand. To address this, a model is proposed in which order quantity depends on a service level policy. This policy can be established under the following conditions:

1. A uniform service level for all items.
2. A differentiated service level by category, based on multicriteria classification.
3. A differentiated service level by item, based on multicriteria classification.

The following notation is used for the formulation of the various mathematical models:

Assumptions

The model is developed for only one period (one month).

Classification ABC does not change during the horizon planning (One year).

Prices and budget are known and do not change during the horizon planning.

Demand is normally distributed, and its standard deviation, in conjunction with service level policies, is used to establish safety stock.

Sets:

I: *Set of farmaceutical products.*

Parameters:

$\boldsymbol{C_i}$: *Cost of item i.*

$\boldsymbol{If_i}$: *Final inventory of item i.*

$\boldsymbol{X_{R+L_i}}$: *The forecasted demand for items i in a time interval equal to R+L, where R is the planning horizon and L is lead time.*

$\boldsymbol{\sigma_{R+L_i}}$: *Estimated standard deviation of the forecast errors over an interval equal to R+L, where,* $\sigma_{R+L_i} = \sigma_1 \sqrt{R+L}$, *and* σ_{1_i} *is the standar deviation of the forecast errors.*

$\boldsymbol{S_i}$: *Ordel Up To Level to item i,* $S_i = X_{R+L_i} + SS_i$, *and* SS_i *depends on the level of service desired.*

B: *Budget by time period.*

Variables

$\boldsymbol{Q_I}$: *Quantity of the item i.*

$\boldsymbol{K_i}$: *Security factor of item i.* K_i *is used to obtain a desired level of service associated to a demand with a Normal probability distribution.* K_i *is unrestricted.*

Model 1. Mathematical model for determining order quantities with an equal service level for all items

Since the variable K_i is unrestricted, it is necessary to replace it with a positive part K_i^+ and a negative part K_i^-, where $K_i^+ \geq 0$ and $K_i^- \geq 0$.

Equation 10, maximizes the total value of pharmaceutical products ordered. This objective function seeks to maximize the quantity of products to order in terms of costs in order to use the maximum possible budget. Equation 11 calculates the quantity to order of each item i, expressed as the difference between the maximum level and the effective inventory. Equation 12 calculates the safety stock for item i using a common k, Equation 13 calculates maximum inventory level of the item i and Equation 14 ensures that the budget is not exceeded. Finally, Equation 15 and Equation 16 guarantee that only one factor takes a value.

Objective Function:

$$max\ z = \sum_{i \in I} C_{i*} Q_i \tag{10}$$

Subject to:

$$Q_i = S_i - if_i,\ \forall i \in I \tag{11}$$

$$SS_i = (K_i^+ - K_i^-) * \sigma_{R+L_i},\ \forall i \in I \tag{12}$$

$$S_i = X_{R+L_i} + SS_i,\ \forall i \in I \tag{13}$$

$$\sum_{i \in I} C_{i*} Q_i \leq B \tag{14}$$

$$K_i^+ \geq K_i^- - M * (1 - Y_i),\ \forall i \in I \tag{15}$$

$$K_i^+ \leq K_i^- + M * Y_i,\ \forall i \in I \tag{16}$$

Model 2. Mathematical model for determining order quantities with a differentiated service level by category.

The previous mathematical model is modified to add a constraint that guarantees an equal service level for each classification.

Sets

Set A: Subset of set I for items classified as A.
Set B: Subset of set I for items classified as B.
Set C: Subset of set I for items classified as C.

The Equation 12 is replaced by the following expressions, adding the variables KA, KB and KC corresponding to an equal level of for items A, B and C respectively (See Equations 17, 18 and 19).

$$SS_i = KA * \sigma_{R+L_i}, \ \forall i \in I \tag{17}$$

$$SS_i = KB * \sigma_{R+L_i}, \ \forall i \in I \tag{18}$$

$$SS_i = KC * \sigma_{R+L_i}, \ \forall i \in I \tag{19}$$

Furthermore, for items classified as A, a value of KA greater than or equal to the K associated with a desired service level P_1. See Equation 20.

$$KA \geq K, \ \forall i \in I \tag{20}$$

Model 3. Mathematical model for determining order quantities with a differentiated service level by item and category

In this case, the K_i can take any value; however, for each item classified as A the service level constraint must be satisfied. Thus, the constraints shown in Equation 18 and Equation 19 are replaced by Equation 21, while Equation 17 and Equation 20 remain the same:

$$SS_i = K_i * \sigma_{R+L_i}, \forall i \in (B, C) \tag{21}$$

Results and Discussion

The proposed methodology was applied to a case study using data provided by a Level II hospital.

Case Study

Consider a healthcare organisation that must manage its pharmaceutical inventories under a budget constraint. The pharmaceutical products in question are medicines used for patient care. Although dispensing these products is not the hospital's primary mission, their availability significantly contributes to the improvement of patients' health conditions. In cases where medicine is unavailable, patients or their companions must take responsibility for acquiring it externally.

This means that while the hospital is not strictly obliged to stock all medicines, it is of utmost importance that these are readily available. The budget is defined annually and allocated based on pricing data, demand figures from previous years, and demand forecasts. Once the budget is established, a tendering process is launched to select a single supplier capable of meeting the hospital's pharmaceutical needs.

Each order has a delivery time of three days, and it is assumed that no bulk purchase discounts apply. The annual budget is divided into 12 monthly periods. The organisation manages 217 stock-keeping units, classified into essential and non-essential items, and seeks to meet policy targets of maintaining a service level above 95% for all essential items.

According to records from the past year, 31.3% of pharmaceutical products are classified as Vital, 36.9% as Essential, and 31.8% as Desirable.

Annual demand data, item costs, and the VED (Vital, Essential, Desirable) classification for each item are not included in this layout; the complete dataset is available upon request from the corresponding author.

Classification ABC multi-criteria

The pharmaceutical products listed in the full dataset (omitted from this layout for length; available upon request) were classified using the TOPSIS method.

The weights W_j were determined by the manager of the relevant department through a pairwise comparison using the multicriteria AHP method developed by Saaty (1990). The AHP weighting procedure (omitted from this layout for length; available upon request).

The results of this process yielded the following values: $W_j = {}_{Demand} = 10.6\%$, $W_j = {}_{Cost} = 26\%$ y $W_j = {}_{VED} = 63.4\%$.

Next, the normalization process was carried out by applying Equation 1. A sample of the results for the first five items is presented in Table 2. For example, for the first item, the normalized demand is obtained by dividing the corresponding value of the demand by the total sum of the squared demands so:

- Demand item 1: 52050
- Total sum of the squared demands: 33273049922,1167
- Normalised demand item 1: 0,00000292

Using the same procedure, normalised Cost and normalised VED were obtained.

Table 2. Normalised Matrix for a Sample of Five Pharmaceutical Products

Relative Importance (Weights)	10,6%	26,0%	63,4%
Criteria / Alternative	Demand	Cost	VED
1	0,00000292	0,00000074	0,00299700
2	0,00000218	0,00000030	0,00199800
3	0,00000227	0,00000037	0,00099900
4	0,00000156	0,00000033	0,00299700
5	0,00000055	0,00000038	0,00199800

The Weighted Normalised Matrix was then calculated by applying Equation 2. The results for the first five items are shown in Table 3. In this step, each value is obtained by multiplying the result of the normalised matrix by its corresponding weight. Therefore, for item 1 and criterion Demand, the value of 0.000000310 is obtained by multiplying 0.00000292 by 10.6%.

Table 3. Weighted Normalized Matrix for a Sample of Five Pharmaceutical Products

Alternative \ Criteria	Demand	Cost	VED
1	0,000000310	0,000000193	0,001898139
2	0,000000231	0,000000079	0,001265426
3	0,000000241	0,000000097	0,000632713
4	0,000000166	0,000000085	0,001898139
5	0,000000059	0,000000098	0,001265426

Based on the results obtained from the Weighted Normalised Matrix, the PIS and NIS were selected for each criterion. These results are presented in Table 4.

Table 4. Positive and negative Ideal Solutions PIS and NIS

Ideal Solution PIS - NIS	Demand	Cost	VED
A+	0,000000310	0,000000002	0,001898139
A-	0,000000000	0,000001952	0,000632713

Based on these results, the positive and negative separations for each item are calculated. The results obtained are shown in Table 5 and Table 6. Below each criterion, the partial results of the squared difference between $(v_i^+ - v_{ij})$ and $(v_i^- - v_{ij})$ are displayed, and in the *Separation* column, the results of Equation 7 and Equation 8 are shown for each of the tables, respectively.

Table 5. Values of positive separation for five pharmaceutical products

Alternative \ Criteria	Demand	Cost	VED	Separation +
1	0,0000000000000000	0,0000000000000362	0,0000000000000000	0,0000001903307487
2	0,0000000000000062	0,0000000000000058	0,0000004003257496	0,0006327130168012
3	0,0000000000000047	0,0000000000000089	0,0000016013029984	0,0012654260199772
4	0,0000000000000206	0,0000000000000068	0,0000000000000000	0,0000001657668051
5	0,0000000000000631	0,0000000000000092	0,0000004003257496	0,0006327130643675

Table 6. Values of negative separation for five pharmaceutical products

Alternative \ Criteria	Demand	Cost	VED	Separation –
1	0,000000000000096	0,000000000003095	0,000001601302998	0,001265427275503
2	0,000000000000053	0,000000000003509	0,000000400325750	0,000632715822370
3	0,000000000000058	0,000000000003442	0,000000000000000	0,000001870906302
4	0,000000000000028	0,000000000003486	0,000001601302998	0,001265427402715
5	0,000000000000003	0,000000000003437	0,000000400325750	0,000632715726167

Finally, the Closeness for each item was calculated using Equation 9. The results for the first five items are shown in Table 7.

Table 7. Results of the Closeness Calculation for Each Item

Alternative	Closeness C*
1	0,99984961
2	0,50000111
3	0,0014763
4	0,99986902
5	0,50000105

With the complete Closeness results, the table was organised from highest to lowest to classify the top 20% of items as A, the next 30% as B, and the remaining 50% as C.

The items classified as A are shown in Table 8.

Table 8. Items classified as A

Code	C*	Code	C*	Code	C*	Code	C*
4	0,99986902	77	0,99976203	135	0,99975718	157	0,99975607
1	0,99984961	65	0,999762	113	0,99975713	167	0,99975606
11	0,9998016	85	0,99976193	155	0,9997571	106	0,99975582
33	0,99977782	70	0,99976186	120	0,99975702	125	0,99975581
30	0,9997776	123	0,99975928	132	0,99975695	185	0,99975572
45	0,99976859	83	0,99975926	69	0,99975692	209	0,99975564
47	0,99976843	74	0,99975895	117	0,99975638	149	0,99975547
58	0,99976793	146	0,99975811	205	0,99975632	97	0,99975543
54	0,99976486	127	0,99975781	195	0,99975627	169	0,99975518
37	0,99976357	73	0,99975763	174	0,99975614	202	0,99975491
80	0,99976292	130	0,99975731	152	0,99975608	164	0,99975439

According to these results, out of the total items classified as A, 44 were found to be Vital, 0 were Essential, and 0 were Desirable.

The items classified as B are shown in Table 9.

Table 9. Items classified as B

Code	C*	Code	C*	Code	C*	Code	C*
191	0,99975415	121	0,9996639	60	0,50000114	175	0,50000112
64	0,99975333	162	0,99964868	91	0,50000114	71	0,50000112
39	0,99975218	61	0,99960374	183	0,50000114	81	0,50000112
158	0,999752	145	0,99960049	14	0,50000114	163	0,50000112
93	0,99975137	172	0,99951088	53	0,50000114	90	0,50000112
98	0,9997512	112	0,99950042	160	0,50000114	103	0,50000112
138	0,99975081	134	0,99943339	15	0,50000113	88	0,50000112
181	0,99974487	34	0,9984453	148	0,50000113	198	0,50000112
27	0,99974451	78	0,50000115	119	0,50000113	193	0,50000112
204	0,99974389	143	0,50000115	22	0,50000113	180	0,50000111
208	0,99974145	41	0,50000115	55	0,50000113	82	0,50000111
184	0,99973493	168	0,50000115	72	0,50000113	75	0,50000111
144	0,99973082	171	0,50000114	151	0,50000113	122	0,50000111
190	0,99973062	40	0,50000114	118	0,50000113	2	0,50000111
128	0,99972993	8	0,50000114	142	0,50000113	50	0,5000011
57	0,9997296	192	0,50000114	214	0,50000113	109	0,5000011

For the items classified as B, 24 were found to be Vital, 41 were Essential, and 0 were Desirable.

The items classified as C are shown in Table 10.

Table 10. Items classified as C

Code	C*	Code	C*	Code	C*	Code	C*	Code	C*
213	0,5000011	161	0,50000101	21	0,00152965	84	0,00151351	76	0,00146909
105	0,5000011	200	0,500001	102	0,00152948	212	0,00151336	20	0,00146073
67	0,5000011	62	0,50000098	140	0,00152945	211	0,00151207	111	0,00146073
131	0,50000109	68	0,50000096	215	0,00152932	12	0,00151045	44	0,00145801
166	0,50000109	107	0,50000096	150	0,0015286	95	0,00150808	124	0,0014519
96	0,50000109	170	0,50000092	13	0,00152436	26	0,0015072	35	0,00143816
154	0,50000108	51	0,50000091	6	0,00152344	25	0,00150068	176	0,00143585
189	0,50000108	114	0,5000009	36	0,00152314	79	0,00149939	38	0,00141882
18	0,50000108	206	0,5000009	94	0,00152201	66	0,00149884	17	0,00141361
186	0,50000107	56	0,5000009	100	0,00152162	104	0,00149772	86	0,00140124
133	0,50000107	178	0,5000009	28	0,00152143	217	0,00149633	179	0,00138674

Table 3 continues on next page

Items classified as C (Cont.)

Code	C*	Code	C*	Code	C*	Code	C*	Code	C*
139	0,50000107	207	0,50000085	52	0,00152122	59	0,00149471	159	0,0013746
216	0,50000107	63	0,50000076	108	0,00152007	210	0,0014933	115	0,00135445
23	0,50000106	182	0,50000074	147	0,00151828	110	0,00149192	196	0,00135251
199	0,50000106	46	0,50000068	89	0,00151817	48	0,00149113	136	0,00134958
5	0,50000105	203	0,50000068	156	0,00151717	9	0,00148352	19	0,00131583
24	0,50000105	29	0,50000054	99	0,00151654	116	0,00148267	129	0,00127796
49	0,50000104	194	0,50000026	92	0,00151594	153	0,00147983	43	0,00125876
31	0,50000104	16	0,0015392	87	0,00151566	197	0,00147737	201	0,00124766
7	0,50000104	137	0,00153119	141	0,00151565	3	0,0014763	42	0,00114514
173	0,50000104	101	0,00153056	165	0,00151565	187	0,00147527	10	0,00113925
188	0,50000104	32	0,00153011	177	0,00151507	126	0,00147359		

For the items classified as C, 0 were found to be *Vital*, 39 were *Essential*, and 69 were *Desirable*.

If these results were compared to those obtained by a single-criterion classification, for the Demand criterion, when ordered from highest to lowest demand, the items classified as A would be 10 Vital, 15 Essential, and 19 Desirable; the items classified as B would be 23 *Vital*, 23 *Essential,* and 23 *Desirable*; and the items classified as C would be 36 *Vital*, 42 *Essential*, and 31 *Desirable*.

When compared to a classification based solely on Cost, when ordered from highest to lowest cost, the items classified as A would be 10 Vital, 15 Essential, and 19 Desirable; the items classified as B would be 23 Vital, 23 Essential, and 20 Desirable; and the items classified as C would be 36 Vital, 42 Essential, and 31 Desirable.

Lastly, when compared to a classification based only on VED, when ordered first by Vital, then Essential, and lastly by Desirable, the items classified as A would be 44 Vital, 0 Essential, and 0 Desirable; the items classified as B would be 25 Vital, 41 Essential, and 0 Desirable; and the items classified as C would be 0 Vital, 39 Essential, and 70 Desirable.

These results show that, when considering other factors, the relative importance of pharmaceutical products changes. For example, when looking at products classified as Vital, 44 were classified as A, 24 as B, and 0 as C, indicating that the perceptions of importance in this case were consistent with the importance given to the VED criterion.

Estimation of Order Quantities via Mathematical Modelling

The results of the multicriteria ABC classification were entered as ordered sets along with the parameters for final inventory, demand forecast, and budget. These data are not included in this layout; the complete dataset is available upon request from the corresponding author.

The proposed models were programmed in AMPL (A Mathematical Programming Language) and solved using the Gurobi solver.

Results of the Model for Determining Order Quantities with an Equal Service Level for All Items - Model 1

The solution obtained by solving the problem using Model 1 presents a set of values for the variables K_i^+ and K_i^-. The values taken by the variable K_i^+ were 2.56 and 0, indicating that it only allows ordering those products that contribute most to maximising the objective function, i.e., the items with the highest value. In this solution, a total of 93 items were assigned a service level of zero, of which 20 were classified as A, 25 as B, and 48 as C. These results are shown in the Table 11.

Results of the Model for Determining Order Quantities with Differentiated Service Levels by Category - Model 2

For this model, minimum service level restrictions were imposed by classification, with 97.5% for items classified as A, 95% for B, and 90% for C.

The solution obtained by solving the model shows that, for items classified as A, the service level exactly matched the minimum value imposed by the restrictions. In this case, a new experiment was conducted, increasing the service levels to 98% for A, 97.5% for B, and 95% for C from 90%. However, the problem became infeasible. This suggests that finding the combination of service level values that performs best for this model may not be easy.

Table 11. Items with service level equal to cero. Model 1.

Code	ABC	Code	VED	Code	VED	Code	VED	Code	VED	Code	VED
31	C	65	A	92	C	126	C	167	A	188	C
32	C	67	C	94	C	128	B	168	B	189	C
38	C	68	C	99	C	133	C	169	A	190	B
39	B	69	A	100	C	135	A	170	C	191	B
40	B	71	B	102	C	136	C	171	B	192	B
41	B	72	B	103	B	137	C	172	B	194	C
44	C	73	A	105	C	139	C	174	A	197	C
45	A	76	C	106	A	141	C	175	B	200	C
46	C	77	A	110	C	150	C	176	C	201	C
47	A	78	B	112	B	152	A	177	C	205	A
50	B	80	A	115	C	153	C	178	C	206	C
53	B	81	B	116	C	154	C	179	C	207	C
54	A	84	C	118	B	157	A	182	C		
58	A	85	A	121	B	162	B	184	B		
59	C	87	C	122	B	165	C	185	A		
64	B	89	C	123	A	166	C	186	C		

Results of the Model for Determining Order Quantities with Differentiated Service Levels by Item and Category - Model 3

Given the results obtained in Model 2, a new model was proposed, aiming to combine Model 1 with Model 2, allowing service levels to take any value above certain permissible limits. These limits were set at 97.5% for items classified as A, 95% for B, and 50% for C. Additionally, an upper bound was imposed for service levels not to fall below 99.5%.

For Model 3, the results showed that for most items, the service level was at the maximum permitted. The other results were as follows: two items classified as A with a service level of 97.5%, one item classified as B with a service level of 98.6%, two items classified as B with a service level of 95%, and eight items classified as C with a service level of 50%.

In all the models presented, the budget was fully utilized; however, Model 3 provided the best result for the data in this case, as it allows the majority of items to be assigned a high service level.

Based on the results obtained and understanding that the goal is to maximize the service level for most items, the objective function of Model 3 was modified to maximize the sum of the K_i factors. The new objective function is presented in Equation 22.

$$max\ z = \sum_{i \in I} K_i \tag{22}$$

Additionally, considering the previously obtained results, the service levels for items classified as A are restricted to a value of 97.5%, with no service level exceeding 99.5%.

The results obtained for this model showed that they are quite similar to those presented in Model 3. A total of 211 items had a service level of 99.5%, one item classified as C had a service level of 76%, and two items classified as B and three items classified as C had a service level of 50%.

In terms of stockout risk, considering that it is calculated as 100% minus the service level, i.e., the probability of a stockout event occurring in a given period, the results in terms of stockout risks for the four models are occasionally presented in Table 12.

The results presented in Table 12, show a better performance for Model 4 in this case study. Considering that every time a stockout occurs, it is up to the patient to manage their pharmaceutical products, it is highly unlikely that this will happen, and in the event that it does, it would only be very likely on five or six occasions.

https://doi.org/10.25100/cdea.v42i84.14855

Table 12. Results of the proposed models in quantity by percentage of stockout risk by classification.

Model	Shortage A		Shortage B		Shortage C	
	Quantity	Shortage Risk [%]	Quantity	Shortage Risk [%]	Quantity	Shortage Risk [%]
Model 1	24	0.5	40	0.5	60	0.5
	20	50	25	50	48	50
Model 2	Infeasible	2.5	Infeasible	5	Infeasible	90
Model 3	42	0.5	62	0.5	100	0.5
	2	2.5	1 y 2	1.4 y 5	8	50
Model 4	44	0.5	63	50	105	50
	-	-	2	50	1 y 3	24 y 50

Conclusions

The inventory control problem for multiple items with stochastic demands and budget constraints was solved using a two-phase hybrid model, combining multi-criteria classification with mathematical modelling. Multi-criteria classification, using the TOPSIS method, allowed pharmaceutical products to be classified, considering not only cost and demand but also their importance according to VED categories. Other criteria were not considered here, as these are fixed and defined during the negotiation, such as the chosen supplier. The Pareto principle was used for classification, with 20% of these items classified as A; in this category, all products considered VITAL were classified. Furthermore, mathematical modelling allowed order quantities to be established for each item, considering budget constraints. A total of four models were developed, seeking to allocate the entire budget while striving to provide the best overall service level. Each model proposes a strategy in which values can be assigned to different service levels to determine order quantities. Of these four models, Model 4, for this case, was found to present favourable results in terms of service levels, while Model 2, under the imposed restrictions, proved infeasible and was therefore not considered for this work. In the proposed case, models 1, 3, and 4 functioned correctly. Although the results of model 4 were better than the other two, due to the very low computation times, these models can be replicated for any real-world instance, and based on the analysis of results, the set of policies that the decision-maker deems best can be selected.

As a subject for future research, it is suggested to apply the methodology proposed here in other cases such as in the management of spare parts inventories or perishable products.

Article funding information

The author declares that no funding was received for the development of this article. It is clarified that the academic software AMPL was used, whose license is free for teaching optimization and logistics courses.

Ethical Implications

The author declares that there are no ethical implications that need to be disclosed in the writing and publication of this article.

Conflict of Interest Statement

The author declares that there is no conflict of interest in the writing and publication of this article.

Author Contribution

Julio César Londoño Ortega

Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration.

Article Provenance Information

The article stems from an idea originating in an undergraduate thesis supervised by me. The development and models presented here differ from those developed in that thesis and are entirely original.

Declaration of the Use of Artificial Intelligence

For the development of this article, the author declares not having used any Artificial Intelligence tools.

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